

## Tilburg University

### A Further Analysis on Strategic Timing of Adoption of New Technologies under Uncertainty

Huisman, K.J.M.; Kort, P.M.

*Publication date:*  
1998

[Link to publication in Tilburg University Research Portal](#)

*Citation for published version (APA):*

Huisman, K. J. M., & Kort, P. M. (1998). *A Further Analysis on Strategic Timing of Adoption of New Technologies under Uncertainty*. (CentER Discussion Paper; Vol. 1998-03). Microeconomics.

#### General rights

Copyright and moral rights for the publications made accessible in the public portal are retained by the authors and/or other copyright owners and it is a condition of accessing publications that users recognise and abide by the legal requirements associated with these rights.

- Users may download and print one copy of any publication from the public portal for the purpose of private study or research.
- You may not further distribute the material or use it for any profit-making activity or commercial gain
- You may freely distribute the URL identifying the publication in the public portal

#### Take down policy

If you believe that this document breaches copyright please contact us providing details, and we will remove access to the work immediately and investigate your claim.

# A further analysis on strategic timing of adoption of new technologies under uncertainty

Kuno J.M. Huisman\* and Peter M. Kort

*Department of Econometrics and CentER,  
Tilburg University, P.O. Box 90153, 5000 LE TILBURG, The Netherlands*

---

## Abstract

In this paper we analyze technology adoption in the context of a duopoly, where the time between adoption and successful implementation is uncertain. This framework is taken from Stenbacka and Tombak, and as such it adds uncertainty to the much cited work of Fudenberg and Tirole.

The analysis is mainly focussed on the case where the firm roles are endogenous. We find that under a certain scenario dispersed adoption timings turn into joint-adoption when firm roles become endogenous. Further, it is shown that for reasonable parameter values it can happen that the profit stream belonging to the preemption equilibrium is that low that both firms are even better off if they both decide to stick to producing with their old technology forever. Another interesting result is that if we have joint adoption in the feedback case with exogenous firm roles, the most reasonable outcome results in delayment of joint-adoption in case that firm roles become endogenous.

*Key words:* Strategic timing; Adoption of new technology

*JEL classification:* O32; O31

---

---

\*Corresponding author. Tel.: (+31)-13-4663176; fax: (+31)-13-4663280; e-mail: huisman@kub.nl

## 1. Introduction

In this paper a duopoly game of timing of adoption of new technologies is analyzed. The model is taken from Stenbacka and Tombak (1994) and it adds uncertainty by incorporating experience effects in the model introduced by Reinganum (1981). There are two identical firms active on an output market. An infinite planning horizon is considered, on which the firms maximize their value and discount at rate  $r$ . At time  $t = 0$  a new technology becomes available and the firms must decide when to adopt that technology. If a firm implements the new technology successfully it makes higher profits. The time between adoption and successful implementation is uncertain and assumed to be exponentially distributed with rate  $\lambda_L$  ( $\lambda_F$ ) for the leader (follower). The investment cost,  $K$ , is decreasing over time, but at an increasing rate:

$$K(t) > 0, \quad K'(t) \leq 0 \quad \text{and} \quad K''(t) \geq 0.$$

There is a first mover advantage in the sense that the gains for being first to successfully implement the new technology are higher than for being second:

$$\pi(s, u) - \pi(u, u) > \pi(s, s) - \pi(u, s) \geq 0,$$

where  $\pi(s, u)$ , for example, is defined as the profit flow in equilibrium for a firm that has successfully implemented the new technology and its competitor has not. The four possible profit flows are assumed to be ranked in the following way:

$$\pi(s, u) > \pi(s, s) \geq \pi(u, u) \geq \pi(u, s) \geq 0.$$

The contents of this paper is as follows. In Section 2 and 3 the open-loop and feedback cases are shortly addressed, thereby correcting some of the results in Stenbacka and Tombak (1994). In the last section we consider the case where the firm roles are endogenous. By doing this we are able to show that there is never an incentive to follow, and identify the situations under which dispersed adoption times and joint-adoption occur.

## 2. Open-loop equilibrium

The open-loop equilibrium  $(T_L^*, T_F^*)$  is equal to the solution of the following system of simultaneous equations (see Stenbacka and Tombak (1994)):

$$e^{-\lambda_L(T_F^* - T_L^*)} = \frac{\frac{\lambda_F}{\lambda_F + r} (\pi(s, s) - \pi(u, s)) - rK(T_F^*) + K'(T_F^*)}{\frac{\lambda_F}{\lambda_F + \lambda_L + r} (\pi(s, s) - \pi(u, s) - \pi(s, u) + \pi(u, u))}, \quad (1)$$

$$e^{-(\lambda_L + r)(T_F^* - T_L^*)} = \frac{\frac{\lambda_L}{\lambda_L + r} (\pi(s, u) - \pi(u, u)) - rK(T_L^*) + K'(T_L^*)}{\frac{\lambda_F \lambda_L}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(s, u) - \pi(u, u) - \pi(s, s) + \pi(u, s))} \quad (2)$$

The following example shows that the effects of increasing the degree of uncertainty (lower values of  $\lambda_L$  and/or  $\lambda_F$ ) and decreasing the advantage of being the first to succeed with implementing the new technology (lower value of  $\pi(s, u) - \pi(u, u) - (\pi(s, s) - \pi(u, s))$ ) on the extent of dispersion between adoption timings (the difference  $T_F^* - T_L^*$ ) are ambiguous. As such it is a counterexample to Stenbacka and Tombak's result that the extent of dispersion is increasing with the degree of uncertainty and with reducing the advantage of being the first successful implementor.

*Example 1.*

We complete the example Stenbacka and Tombak use to illustrate case (b) of their Proposition 1. There is a linear inverse demand function,  $p = a - q_1 - q_2$ , constant marginal costs,  $c$ , an innovation that reduces the marginal cost to  $c - \varepsilon$ , and Cournot competition. In this situation we have

$$\begin{aligned}\pi(s, u) &= \frac{1}{9}(a - c + 2\varepsilon)^2 \\ \pi(s, s) &= \frac{1}{9}(a - c + \varepsilon)^2 \\ \pi(u, u) &= \frac{1}{9}(a - c)^2 \\ \pi(u, s) &= \frac{1}{9}(a - c - \varepsilon)^2\end{aligned}$$

We take the investment cost  $K$  at time  $t$  equal to  $K(t) = 100e^{-0.1t}$ , the discount rate is  $r = 0.05$ . Solving equations (1) and (2) simultaneously for given values of  $a, c, \varepsilon, \lambda_L$  and  $\lambda_F$  gives the values for  $T_F^*$  and  $T_L^*$ . We take  $a - c = 4$  and let  $\lambda_L, \lambda_F$  and  $\varepsilon$  vary. The results are summarized in the next table.

$\varepsilon$	$\lambda_L$	$\lambda_F$	$T_L^*$	$T_F^*$
1	0.15	0.15	22.868	22.868
1	0.25	0.15	21.253	23.605
1	0.25	0.25	21.872	21.872
2	0.25	0.25	13.556	14.067

Comparing the second and the first scenario we see that increasing the uncertainty ( $\lambda_L \downarrow$ ) leads to a **decreased** extent of dispersion of the adoption timings. If we compare the third and the second scenario we see that an increase of the degree of uncertainty leads to an increase of the extent of dispersion. Hence it is unclear whether an increase of the degree of uncertainty leads to an increase or decrease of the extent of dispersion.

By comparing the third and fourth scenario we see that an increase of  $\varepsilon$  (the cost reduction from successful implementation) results in an **increase** of the extent of dispersion.

### 3. Feedback equilibrium

Comparing the feedback equilibrium timings  $(T_L^{**}, T_F^{**})$  to those of the open-loop case the following proposition can be formulated (see Stenbacka and Tombak (1994)):

*Proposition 1.*

*The feedback equilibrium timings are more dispersed than those of the open-loop case, i.e.*

$$T_L^{**} < T_L^* < T_F^* < T_F^{**}.$$

Intuitively this result is understandable since the reaction function of the follower, given by equation (1), is such that  $\frac{\partial T_F}{\partial T_L} < 0$ . Hence, the leader knows that when it adopts earlier the follower will adopt later. This strategic interaction, which is present in the feedback equilibrium, thus leads to a larger expected time interval on which the leader collects first implementor profits. Compared to open-loop, this gives an extra incentive for the leader to adopt earlier. This tendency to adopt earlier, however, is tempered by the increase in investment costs. A more extensive proof than the one provided by Stenbacka and Tombak (1994) can be found in Appendix A.

In Stenbacka and Tombak (1994) it is shown that, contrary to Fudenberg and Tirole (1985), a joint-adoption equilibrium exists<sup>1</sup>. Furthermore, Stenbacka and Tombak argue that for identical firms the joint-adoption time in the feedback case ( $T_S^{**} = T_L^{**} = T_F^{**}$ ) is smaller than the joint-adoption time in the open-loop case ( $T_S^*$ ):

$$T_S^{**} < T_S^*. \quad (3)$$

Their mathematical derivation of this statement is correct, but there is no economic meaning. The first reason is that, when firms are identical, joint-adoption in the open-loop as well as the feedback equilibrium can not occur for the same parameter set. As argued by Stenbacka and Tombak (1994), if the feedback equilibrium exhibits joint-adoption, there does not exist an open-loop equilibrium and if the open-loop equilibrium is of the joint-adoption type, there is dispersion in the feedback case (direct result of Proposition 1). Second, when the sets of parameters differ equation (3) need not hold since Stenbacka and Tombak derive equation (3) by assuming that the parameter set is the same for open-loop and feedback. Given the parameter set and the resulting adoption time  $T_S^{**}$  of the feedback case, it should not be difficult to find a different parameter set which results in an adoption time  $T_S^*$  under open-loop such that  $T_S^* < T_S^{**}$ . We conclude that we cannot say that simultaneous adoption occurs earlier under feedback than under open-loop.

---

<sup>1</sup>Notice that here we consider exogenous firm roles. Fudenberg and Tirole do find joint-adoption equilibria in the case of endogenous firm roles.

#### 4. Endogenous firm roles

To reduce the number of scenarios considerably we assume, from now on, that the firms are identical. We analyze this problem by deriving Fudenberg and Tirole (1985) look alike figures. Define the following three functions

$$L(t) = EV_L(t, T_F(t)), \quad (4)$$

$$F(t) = EV_F(t, T_F(t)), \quad (5)$$

$$M(t) = EV_i(t, t), \quad i = L, F, \quad (6)$$

where  $EV_L(T_L, T_F)$  ( $EV_F(T_L, T_F)$ ) is the expected total discounted profit to the leader (follower) when the leader adopts at time  $T_L$  and the follower at time  $T_F$ <sup>2</sup>. The function  $L(t)$  ( $F(t)$ ) is equal to the expected discounted value at time zero of the leader (follower) when the leader invests at time  $t$ .  $M(t)$  resembles the value of the firm when there is joint-adoption at time  $t$ . The best reply adoption time of the follower when the leader invests at time  $t$  is denoted by  $T_F(t)$ .

First, observe that there exists a  $\hat{T}$  such that for every  $t \geq \hat{T}$  the equilibrium involves joint-adoption:

$$\hat{T} = \min \{t | T_F(t) = t\}.$$

If the investment cost is such that it is optimal for both firms to adopt at time zero we have nothing to prove. In the more interesting case, preemption of the leader at time zero will lead to a dispersed equilibrium. We know that the extent of dispersion decreases as a function of the preemption time of the leader, because expression (1) implies that  $\frac{\partial T_F}{\partial T_L} < 0$ . Since by definition  $T_L \leq T_F$  this implies the existence of  $\hat{T}$ .

The above story implies that before time  $\hat{T}$  joint-adoption is not optimal, i.e.

$$\begin{aligned} M(t) &< L(t), & t < \hat{T}, \\ M(t) &< F(t), & t < \hat{T}, \end{aligned}$$

and that after time  $\hat{T}$  joint-adoption is optimal:

$$L(t) = F(t) = M(t), \quad t \geq \hat{T}. \quad (7)$$

Second, the expected payoff of the follower increases when the leader adopts later. Hence,  $F$  is increasing in  $t$ .

Using the definitions (4)-(6), we define (see also Section 3 for the meaning of  $T_L^{**}$ ):

$$\begin{aligned} T_L^{**} &= \arg \max_{t \in [0, \hat{T}]} L(t), \\ T_C^* &= \arg \max_t M(t). \end{aligned}$$

---

<sup>2</sup>The functions  $EV_L(\cdot, \cdot)$  and  $EV_F(\cdot, \cdot)$  are defined by Stenbacka and Tombak (1994)'s equations (1) and (2).

Stenbacka and Tombak call adoption time  $T_C^*$  a cooperative one. However, as we see later, it can also occur as a non-cooperative outcome when firm roles are made endogenous. Since  $T_C^*$  maximizes the sum of the profits of the firms, it also maximizes the value of the firm under joint adoption.

The analysis above implies that there are three possible cases. Case A is characterized by  $T_L^{**} < \hat{T}$  and  $L(T_L^{**}) > M(T_C^*)$ . Case B by  $T_L^{**} < \hat{T}$  and  $L(T_L^{**}) \leq M(T_C^*)$  and case C by  $T_L^{**} = \hat{T} (= T_S^{**})$ . The first two cases also occur in Fudenberg and Tirole (1985). What is new is case C. Occurrence of case C follows from the fact that a joint-adoption equilibrium exists in the case of a feedback information structure with exogenous firm roles (Section 3). This equilibrium does not occur in Fudenberg and Tirole (1985), which implies that they do not have a case C. In Figures 1, 2 and 3 we have plotted the value functions  $L(t)$ ,  $F(t)$  and  $M(t)$  for each case.

In cases A and B time  $S$  is defined as:

$$S = \min \{t \in [0, T_L^{**}] | L(t) = F(t)\}.$$

In equilibrium of case A there will be one firm that preempts at time  $S$  and the other will react by adopting at time  $T_F(S) > S$ . This implies that adoption timings are more dispersed and that there is rent equalization in this equilibrium.

For the example presented in Section 2 we arrive in Case A if we take  $\pi(s, u) = 6$  and  $\lambda_L = \lambda_F = 0.25$ . We found that, roughly, for discount rates larger than 3% it holds that  $L(S) = F(S) < \frac{\pi(u, u)}{r}$  (see Figure 1). Economically this means that both firms can do better by sticking to producing with their old technology forever. Still, strategic interactions drive them to the preemption equilibrium just mentioned.

To analyze case B, first define time  $R$ :

$$R = \min \{t \in [\hat{T}, T_C^*] | M(t) = L(T_L^{**})\}.$$

In case B there are multiple equilibria, which can be split up in two classes. The first class consists of the  $(S, T_F(S))$  diffusion equilibrium. The second class is a continuum of joint-adoption outcomes indexed by the date of adoption  $t \in [R, T_C^*]$ . So in this case Stenbacka and Tombak's cooperative outcome, adoption at time  $T_C^*$ , is also a non-cooperative one.

In case C there is also a continuum of joint-adoption equilibria, but no diffusion equilibrium. These equilibria are again indexed by the date of adoption  $t \in [\hat{T}, T_C^*]$ . As already mentioned, this case can occur, because there exist parameter sets for which the feedback equilibrium in the model with exogenous firm roles exhibits joint-adoption at time  $T_S^{**3}$ . Note that in Fudenberg and Tirole (1985) and in Reinganum (1981) this case can not occur. Reinganum proves that

---

<sup>3</sup>Note that Stenbacka and Tombak derive on page 403 that  $T_C^* > T_S^{**}$ .

in the feedback equilibrium of her model with exogenous firm roles it is true for all parameter sets that  $T_L^{**} < T_F^{**4}$ .

We summarize the analysis by extending Fudenberg and Tirole's Proposition 2:

*Proposition 2.*

(A) If  $T_L^{**} < \hat{T}$  and  $L(T_L^{**}) > M(T_C^*)$  there exists a unique equilibrium distribution over outcomes. With probability one-half, firm one adopts at time  $S$  and firm two adopts at  $T_F(S)$ , and with probability one-half the roles of the firms are reversed. Thus the equilibrium exhibits diffusion; and with probability one the adoption dates are  $S$  and  $T_F(S)$ .

(B) If  $T_L^{**} < \hat{T}$  and  $L(T_L^{**}) \leq M(T_C^*)$  two classes of equilibria exist. The first class is the  $(S, T_F(S))$  diffusion equilibrium. The second class is a continuum of joint-adoption outcomes indexed by the date of adoption  $t \in [R, T_C^*]$ .

(C) If  $T_L^{**} = \hat{T}$  one class of joint-adoption equilibria exists. The class is again indexed by the date of adoption  $t \in [\hat{T}, T_C^*]$ .

Fudenberg and Tirole (1985) prove that the probability of a mistake in the diffusion equilibrium, i.e. that both firms together adopt at time  $S$  which leads to very low profits, is zero. They derive the following properties of the equilibria, which also hold in this extended version of the Reinganum-Fudenberg-Tirole model.

*Proposition 3.*

In equilibrium the payoffs of the two firms are equated. The equilibria are Pareto comparable to the equilibria of the model with exogenous firm roles (from the firms' point of view).

*Proposition 4.*

All joint-adoption equilibria, if they exist, Pareto-dominate the outcomes for the model with exogenous firm roles, which Pareto-dominate the diffusion equilibrium.

*Proposition 5.*

Joint-adoption equilibria are Pareto-ranked by their date of adoption, later adoption being more efficient from the firm's point of view.

The implication of Proposition 5 is that in cases B and C the most reasonable outcome to expect is the joint-adoption at time  $T_C^*$ , because it Pareto-dominates all other equilibria.

At the end of this section we compare our findings with the main conclusion

---

<sup>4</sup>There is one exception, if the investment cost at time zero is low enough there will be joint-adoption at time zero.



of Stenbacka and Tombak (1994), i.e. "when firm roles are made endogenous we find that if there is an incentive to lead (follow) the equilibrium timings are more (less) dispersed". In cases A and B there is an incentive to lead, because from (4), (5) and the Figures 1 and 2 we obtain that

$$EV_L(T_L^{**}, T_F^{**}) > EV_F(T_L^{**}, T_F^{**}).$$

For the diffusion equilibria in cases A and B the result of Stenbacka and Tombak indeed holds, because preemption leads to earlier adoption for the leader, which in turn leads to later adoption for the follower since  $\frac{\partial T_F}{\partial T_L} < 0$ . However, the most reasonable outcome in case B is joint-adoption at time  $T_C^*$  (Proposition 5). We conclude that in case B we have a diffusion equilibrium in the feedback case and joint-adoption in case the firm roles are made endogenous, so that here the opposite happens from what is stated in Stenbacka and Tombak's result.

In case C there is joint-adoption at time  $T_S^{**} = \hat{T}$  in the feedback case with exogenous firm roles, which implies that there is no incentive to become leader or follower. The most reasonable outcome under endogenous firm roles is joint-adoption at time  $T_C^*$ . Hence, in this case making firm roles endogenous leads to later adoption.

Taking all cases together we see that there is never an incentive to follow, so that maturation equilibria are absent (see Dutta et al. (1995)) and the parts between brackets can be dropped in Stenbacka and Tombak's result.

## A Proof of proposition 1

The first-order condition for interior solutions for the leader in the feedback game is

$$\begin{aligned} & \frac{\lambda_L}{\lambda_L + r} (\pi(u, u) - \pi(s, u)) \\ & - \frac{\lambda_F e^{-r(T_F - T_L^{**})}}{\lambda_F + r} \frac{\partial T_F}{\partial T_L} (\pi(s, s) - \pi(s, u)) \\ & + \frac{\lambda_F e^{-(\lambda_L + r)(T_F - T_L^{**})}}{\lambda_F + \lambda_L + r} \frac{\partial T_F}{\partial T_L} (\pi(s, s) - \pi(s, u)) \\ & - \frac{\lambda_L \lambda_F e^{-(\lambda_L + r)(T_F - T_L^{**})}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(s, s) - \pi(s, u)) \\ & + \frac{\lambda_F \left( \lambda_L - (\lambda_L + r) \frac{\partial T_F}{\partial T_L} \right) e^{-(\lambda_L + r)(T_F - T_L^{**})}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(u, s) - \pi(u, u)) \\ & + rK(T_L^{**}) - K'(T_L^{**}) = 0. \end{aligned} \tag{8}$$

Please note that we corrected for the (two) sign mistakes in Stenbacka and Tombak's equation (11). In the open-loop case the first order condition is given

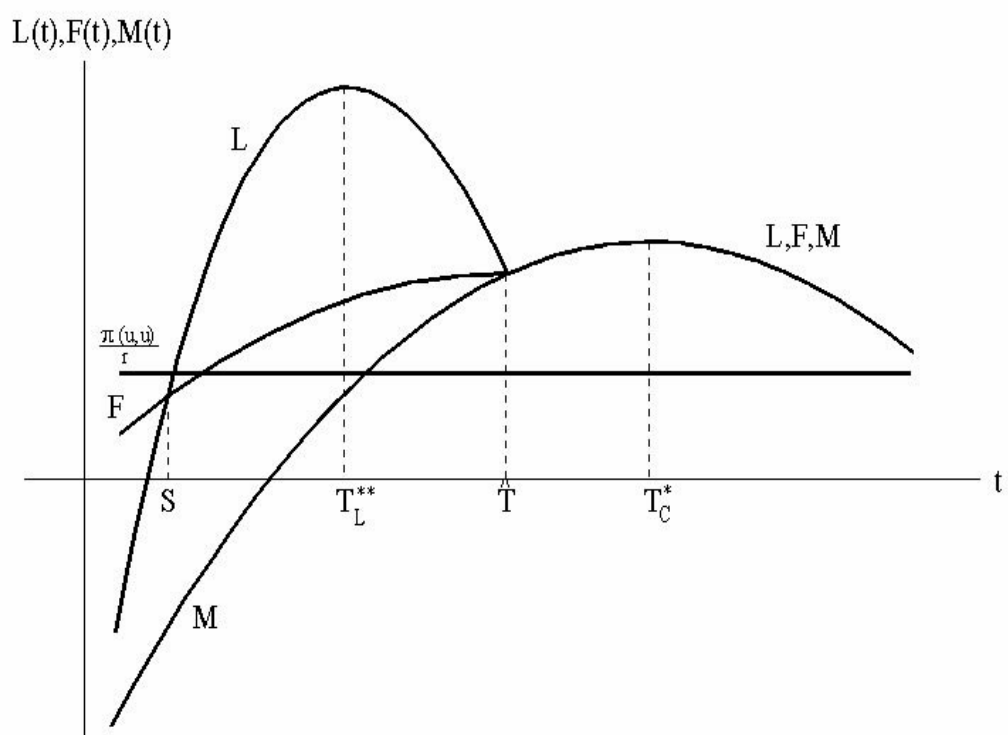


Figure 1: Case A.

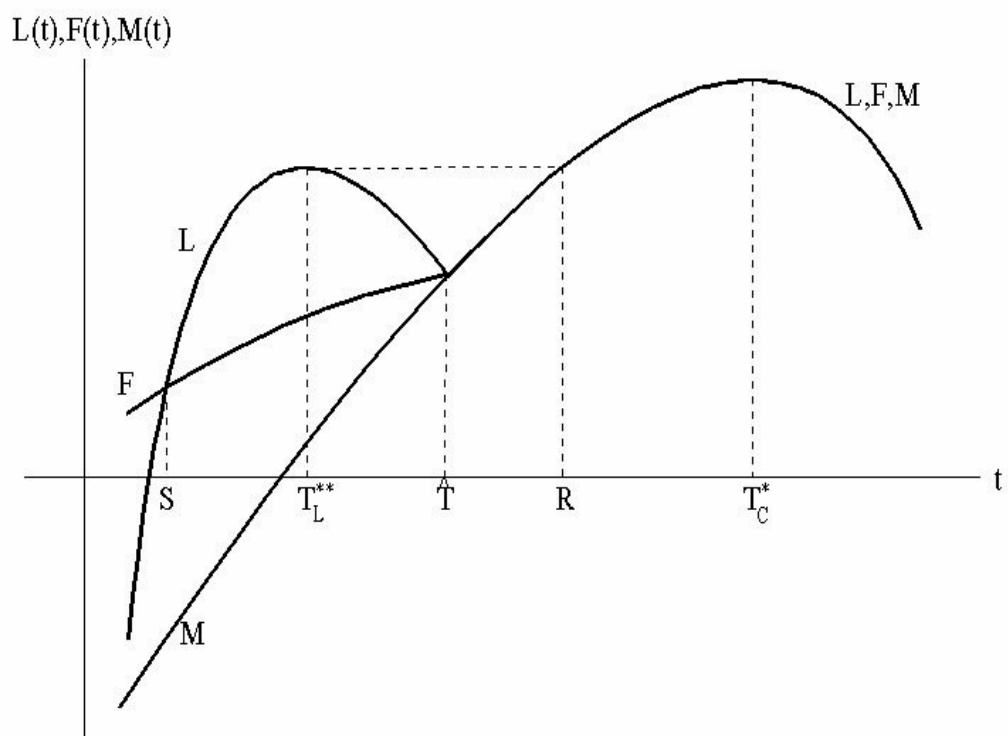


Figure 2: Case B.

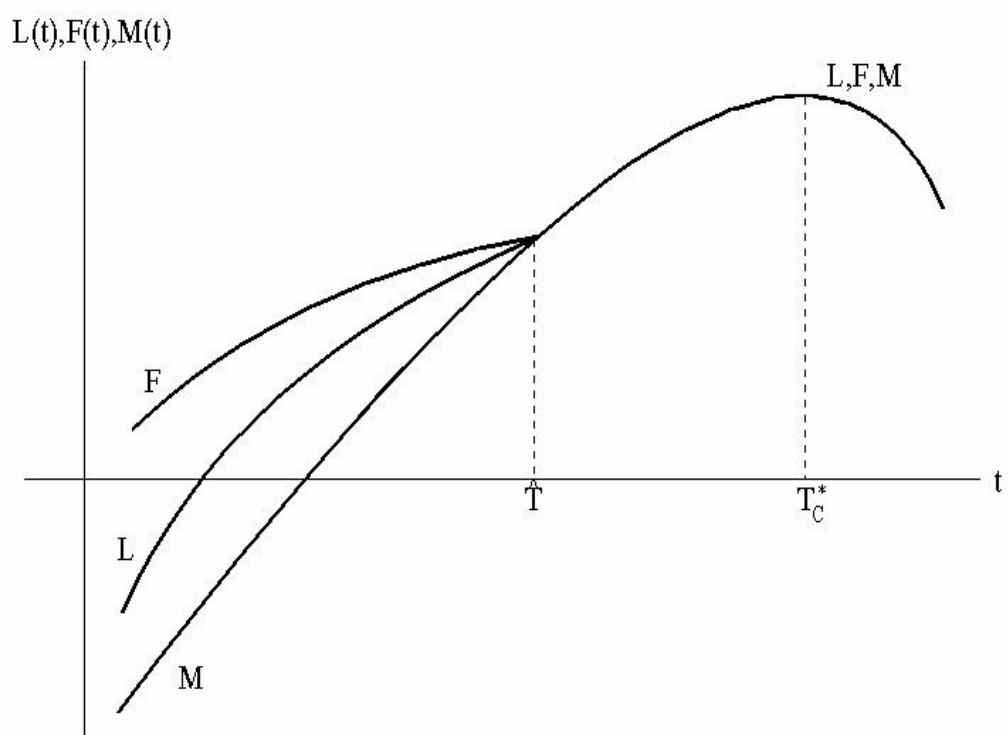


Figure 3: Case C.

by the following rewritten version of equation (2):

$$\begin{aligned}
& \frac{\lambda_L}{\lambda_L + r} (\pi(u, u) - \pi(s, u)) \\
& - \frac{\lambda_L \lambda_F e^{-(\lambda_L + r)(T_F - T_L^*)}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(s, s) - \pi(s, u)) \\
& + \frac{\lambda_F \lambda_L e^{-(\lambda_L + r)(T_F - T_L^*)}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(u, s) - \pi(u, u)) \\
& + rK(T_L^*) - K'(T_L^*) = 0.
\end{aligned} \tag{9}$$

Now, define the following function:

$$\begin{aligned}
f(T_L) &= \frac{\lambda_L}{\lambda_L + r} (\pi(u, u) - \pi(s, u)) \\
& - \frac{\lambda_L \lambda_F e^{-(\lambda_L + r)(T_F - T_L)}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(s, s) - \pi(s, u)) \\
& + \frac{\lambda_F \lambda_L e^{-(\lambda_L + r)(T_F - T_L)}}{(\lambda_L + r)(\lambda_F + \lambda_L + r)} (\pi(u, s) - \pi(u, u)) \\
& + rK(T_L) - K'(T_L).
\end{aligned} \tag{10}$$

From (9) and (10) it follows that

$$f(T_L^*) = 0. \tag{11}$$

Provided that the second order condition holds (cf. Stenbacka and Tombak (1994), page. 409), it can be shown that

$$f'(T_L) < 0. \tag{12}$$

Furthermore, observe that (8) can be written into

$$\begin{aligned}
& f(T_L^{**}) - \frac{\lambda_F e^{-r(T_F - T_L^{**})}}{\lambda_F + r} \frac{\partial T_F}{\partial T_L} (\pi(s, s) - \pi(s, u)) \\
& + \frac{\lambda_F e^{-(\lambda_L + r)(T_F - T_L^{**})}}{\lambda_F + \lambda_L + r} \frac{\partial T_F}{\partial T_L} (\pi(s, s) - \pi(s, u)) \\
& - \frac{\lambda_F e^{-(\lambda_L + r)(T_F - T_L^{**})}}{\lambda_F + \lambda_L + r} \frac{\partial T_F}{\partial T_L} (\pi(u, s) - \pi(u, u)) = 0.
\end{aligned} \tag{13}$$

Since the sum of the three terms is negative, it follows from (13) that

$$f(T_L^{**}) > 0. \tag{14}$$

Now (11), (12) and (14) imply that  $T_L^{**} < T_L^*$ , meaning that in the feedback case the firm will adopt earlier.

## **Acknowledgments**

The authors thank Aldo Rustichini for his comments.

## **References**

- Dutta, P.K., S. Lach and A. Rustichini, 1995, Better late than early: Vertical differentiation in the adoption of a new technology, *Journal of Economics and Management Strategy*, 4, 563-589.
- Fudenberg, D. and J. Tirole, 1985, Preemption and rent equalization in the adoption of new technology, *Review of Economic Studies*, LII, 383-401.
- Reinganum, J., 1981, On the diffusion of new technology: A game theoretic approach, *Review of Economic Studies* XLVIII, 395-405.
- Stenbacka, R. and M.M. Tombak, 1994, Strategic timing of adoption of new technologies under uncertainty, *International Journal of Industrial Organization*, 12, 387-411.